

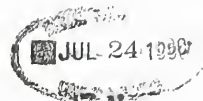
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Money in Search Equilibrium

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1. Introduction

Some economists attribute fluctuations in unemployment to misperceptions of prices and wages. Others attribute such fluctuations to lags in adjustment of prices and wages (including staggered contracts). It seems to be a shared view that there would be no macroeconomic unemployment problems if prices and wages were fully flexible and correctly perceived. This paper continues the examination of a third cause of macro unemployment problems - the difficulty of coordination of trade in a many person economy. That is, once one drops the fictional Walrasian auctioneer and introduces trade frictions, one can have macro unemployment problems in an economy with correctly perceived, flexible prices and wages. This proposition is analyzed in a model where money plays a critical role in coordinating transactions.

To model the transactions role of money, it seems natural to use a continuous time model where transactions occur at discrete times.¹ This

*This paper and my earlier one (1982) were presented as the Fisher-Schultz lecture at the Amsterdam Econometric Society Meetings in 1981. For a more leisurely presentation of some of the same material and a general discussion of search equilibrium, see my forthcoming Wickseil Lectures. I am grateful for useful discussions with J. Mirrlees and C. Wilson, research assistance by M. Whinston, and research support from NSF.

¹For an example of a discrete time macro model based on a continuous time micro model, see Akerlof (1973). For an example of a continuous time model with continuous transactions, see Jovanovich (1982).

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approach conforms with much of common experience and avoids the need to find an appropriate discrete time constraint on individual transactions within a period. It also seems natural to require search for all transactions, rather than closing a partial equilibrium search model with frictionless competitive markets.

This paper studies a simple general equilibrium search model where money is used for all transactions (no barter or credit). The model considers only steady state rational expectations equilibria. Individual production decisions determine the flow of goods into inventories for trade. The aggregate flow of transactions depends on the size of inventories and the stock of individuals with money who are trying to purchase. Individuals experience the arrival of trading opportunities as Poisson processes with parameters consistent with the aggregate flow of transactions. When a buyer and a seller meet, they negotiate a price for the sale of one unit of the indivisible consumer good. It is assumed that negotiations are always successful and that the negotiated price divides evenly the utility gain from completing the transaction. Thus, individuals produce, sell their output, and then use the money to buy output from others.

For a given constant nominal money supply, it is found that there are multiple equilibria, with equilibrium at a higher level of production associated with lower prices and a higher real money supply. That is, an economy with this structure of transaction costs has multiple natural rates of unemployment. This suggests that in the presence of macro shocks, macro policy is needed to keep the economy from spending long periods of time in a low level equilibrium. Even with price controls, and so a fixed real money supply, there are multiple equilibria.

A further finding is that none of the internal equilibria of the economy with endogenous prices are efficient relative to policies which could vary the incentive to produce and the real money supply. We do not examine policies to

affect those variables here, since this would take us out of the simple steady state equilibrium.

The paper begins with a simple overview of the model (Section 2). The trade process is presented next (Section 3), then the production process (Section 4), and then optimal individual production decisions (Section 5). The rule for price determination and analysis of equilibrium are in Section 6. Equilibria with price controls are in Section 7. Local efficiency is discussed in Section 8. The model is discussed, extended, and given an alternative interpretation in the concluding sections.

2. Overview

Before presenting the precise specification of the model, we start with a parable which describes the basic mechanism. Consider a tropical island with many people. Individuals walk along the beach looking for palm trees. Each tree has one nut. Trees differ in height and there is a disutility to climbing. Therefore, having found a tree, an individual must decide whether to climb it. There is a taboo on eating nuts one has picked oneself. Thus, having climbed a tree, an individual must engage in trade to enjoy any consumption. The assumed taboo plays the role of the advantage of specialization and trade over self-sufficiency in a modern economy. For analytical convenience, assume that an individual never chooses to climb a second tree when he has a nut in inventory.

There is a fixed quantity of fiat money on this island. Having picked a nut, the individual looks for someone with money to whom to sell the nut. By assumption there is no barter. This represents the fact that the problem is really finding a nut to one's taste rather than any nut. For simplification there is no credit. When a seller finds a buyer, they negotiate a mutually agreeable price. This mutually agreeable price is such that the gain to the



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seller from the transaction (rather than waiting for the next buyer) is equal to the gain to the buyer (rather than waiting for the next seller). Having sold his inventory, the individual goes shopping with the money received. After completing a purchase, the individual goes back to searching for short trees.

In this setting, one can consider rational expectations steady state equilibria. That is, we consider equilibria where (1) the production decision is individually optimal given the correctly perceived parameters of the production and trade processes and (2) the pricing rule is based on the correct evaluation of the gains in expected utility from carrying out transactions. We find that the economy has multiple equilibria and that these are inefficient relative to policies which can directly control production incentives and the real money supply.

3. Trade Process

We begin by describing the technology controlling the matching of buyers and sellers. This is made up of two parts (1) a determinate aggregate trade function relating the flow number of trades to the stocks of buyers and sellers and (2) Poisson processes giving the stochastic rates of transactions for each individual.

Assume there are m individuals with money trying to buy the commodity and e individuals employed in the trade process with inventories they are trying to sell. (With n individuals in aggregate, $n-e-m$ of them are searching for production opportunities.) Then, the rate of completion of transactions will be written as $f(e,m)$, which is assumed to be twice continuously differentiable and to have well behaved isoquants. To have the aggregate rate of transactions depend on the numbers of would be buyers and sellers is a natural formulation of a transactions technology. In the simplified setting described here, some

individuals ($n-m$) are liquidity constrained and so unable to purchase. (Even with credit, we would expect to find some individuals constrained by available purchasing power.) Apart from the number of liquidity constrained individuals, there is no attempt to capture the magnitude of aggregate demand, which depends on many additional factors.

We make a number of assumptions about the trade technology. First, we assume positive marginal products provided a positive number on the other side of the market

$$f_e(e, m) > 0, f_m(e, m) > 0 \text{ when } m > 0, e > 0 \quad (1)$$

Second, we make the assumption of increasing returns to scale

$$ef_e + mf_m > f \quad (2)$$

Increasing returns to scale are very plausible at low levels of activity. They also become plausible at higher levels once one recognizes the geographic dispersion of buyers and sellers and thus the gain from increased transaction locations as the density of traders grows. In a business cycle context, with a fixed infrastructure of trading capacity, there is increased short run profitability from increased use of retail trade facilities.² Third, we assume that the returns to scale are not so large that the average rate of transactions rises with an increase in traders on that side of the market. Differentiating f/e with respect to e and f/m with respect to m , we have

$$f \geq ef_e, f \geq mf_m \quad (3)$$

Fourth, we assume that f/m goes to zero as m rises without limit or e goes to zero; while f/m has a finite limit if e rises without limit or m goes to zero.

Symmetric assumptions are made for f/e .

²For an argument that macro problems are inherently connected with increasing returns to scale see M. Weitzman (1981).

To go with this description of aggregate outcomes we need a consistent description of the individual experience of the trade process. We assume that each seller experiences the arrival of buyers as a Poisson process with arrival rate b and each buyer experiences the arrival of sellers as a Poisson process with arrival rate s . Individuals view these rates as parameters beyond their control. Thus, we are ignoring systematic search, search intensity, advertising, and reputation as determinants of the outcome of the search process. The results of this analysis seem robust to inclusion of these elements, since optimization over additional control variables would still leave the profitability of production for trade an increasing function of potential trading partners.

For micro-macro consistency, the sum of individual experiences must equal the aggregate experience, which has been assumed to be nonstochastic. Thus we have

$$be = sm = f(e, m) \quad (4)$$

That is, the arrival rate of buyers times the number of sellers equals the arrival rate of sellers times the number of buyers equals the aggregate rate of meetings. Since there are no further matching problems and the negotiation process is assumed to be instantaneous and always successful, the rate of meetings equals the rate of transactions.

4. Production

Rather than modelling production as a continuous process with varying intensity, we follow the slightly simpler course of viewing production as instantaneous, with the search for production opportunities as time consuming. Thus, each individual not engaged in buying or selling has the possibility of finding a production opportunity. The arrival of production opportunities is a Poisson process with arrival rate a . Each opportunity represents one indivisible

unit of output available for sale. Each opportunity involves a labor cost, c , which is an independent draw from the exogenous distribution $G(c)$. It is assumed that there is a strictly positive minimal cost of production $\underline{c}$. That is, the support of G is bounded below by $\underline{c}$. For convenience we also assume that the support is connected and unbounded and G is differentiable.

Only those not engaged in buying or selling are searching for production opportunities. If all projects costing less than c^* are taken, the rate of change in the stock of inventories satisfies

$$\dot{e} = aG(c^*)(n-e-m) - f(e,m) \quad (5)$$

That is, inventories grow from production by searchers who accept all projects costing less than c^* and shrink from sales. We will be concentrating on steady state equilibria where $\dot{e}$ is zero. We will also be considering only equilibria with constant prices. Then, the number of shoppers, m , does not change since a completed transaction converts a buyer into a searcher for production opportunities and a seller into a buyer.

5. Individual Choice

We now model individual choice of c^* , the cutoff cost of production opportunities undertaken. We shall consider only steady states where b and s do not vary. With a constant price level, the sale of one unit is just sufficient to finance the purchase of one unit. Thus money can be thought of as tickets, each good for a single purchase. Individuals are either in a position to buy (i.e. with enough money for a unit of purchase), in a position to sell (with inventory), or neither of the above. We denote the three states by e , m , and u , mnemonics for employed in the trade sector, with money, and unemployed in the trade sector. The disutility (labor cost) of a completed production opportunity

is c . The utility of consumption coming from a completed purchase is denoted u . It is assumed that the individual has a positive discount rate r , lives forever, and seeks to maximize the expected present discounted value of consumption utilities less production disutilities.

We denote the expected present discounted values of lifetime utility conditional on currently being employed in selling, having money to buy, and being in neither of these states as W_e , W_m , and W_u . Since the optimal cutoff rule for production is to undertake any project that costs less than the gain in expected utility from undertaking a project,

$$c^* = W_e - W_u \quad (6)$$

To determine c^* , we use a dynamic programming framework. The rate of discount times the value of being in a position is equal to the sum of expected gains from instantaneous utility and from a change in status. Thus we have the three value equations

$$rW_e = b(W_m - W_e) \quad (7a)$$

$$rW_m = s(u + W_u - W_m) \quad (7b)$$

$$rW_u = a \int_0^{c^*} (W_e - W_u - c) dG(c) \quad (7c)$$

That is, those with inventory wait for buyers; those with money, for sellers; and those with neither, for production opportunities.

Subtracting the equations pairwise and using (6) we have

$$rc^* = b(W_m - W_e) - a \int_0^{c^*} (c^* - c) dG \quad (8a)$$

$$(r+b)(W_m - W_e) = s(u + W_u - W_m) \quad (8b)$$

$$(r+s)(W_u - W_m) = a \int_0^{c^*} (c^* - c) dG - su \quad (8c)$$

Substituting from (8b) and (8c) in (8a) we have an implicit equation for c^* :

$$(r+b)(r+s)c^* = bsu - (r+b+s) a \int_0^{c^*} (c^*-c) dG \quad (9)$$

Defining t by

$$t \equiv \frac{bs}{(r+b+s)} = \frac{f^2}{rem + f(e+m)} \quad (10)$$

we can rewrite (9) as

$$(r+t)c^* = tu - a \int_0^{c^*} (c^*-c) dG \quad (11)$$

Since r , a , and G are fixed throughout the subsequent discussion, we write the solution of (11) as $c^*(t)$. Differentiating with respect to t , we have

$$(r+t+aG(c^*))c^{*'} = u - c^* \quad (12)$$

which is positive by (11): at positive interest, no one would accept a project with lower eventual utility than the current disutility cost. Since t is an increasing function of both b and s (when they are positive) so, too, is c^* . From (10) we could consider iso- t loci in the (e, m) plane to examine the effect of shifts between buyers and sellers on the profitability of production.

To interpret t , consider an individual with a unit of inventory. Then, the expected present discounted value of utility from the sale of that unit followed by the purchase of a unit of consumption is³ $bsu/(r+b)(r+s)$. In addition, completion of sale and purchase frees the individual to pursue production opportunities, which has value W_u at the date of consumption. The expected present discounted value of utility from a position with a unit of inventory, W_e , can be written (from (6), (7c), and (11)) as

$$\begin{aligned} rW_e &= r(c^* + W_u) = rc^* + a \int_0^{c^*} (c^*-c) dG \\ &= t(u - c^*) \end{aligned} \quad (13)$$

³ If the purchase of a good were the end of economic activity, the value of having a unit in inventory would equal W_e in (7) with W_u set equal to zero.

Another way of seeing (13) is to notice that the individual is indifferent between drawing from the distribution of costs $G(c)$ with arrival rate a and having instant access to a project that costs c^* . With the latter structure, the gain from a purchase plus restocking of inventories is $(u-c^*)$ which occurs repeatedly, the gap between any two consumptions having expected present discounted value $bs/(r+b)(r+s)$. Taking the sum of discounted consumption plus restockings we have

$$(u-c^*) \sum_{i=1}^{\infty} \left(\frac{bs}{(r+b)(r+s)} \right)^i = (u-c^*)t/r \quad (14)$$

Of course, the constancy of the discount rate is critical for this simple formulation.

6. Price Determination

If prices are exogenous and the real money supply fixed, equilibria are the solutions to $\dot{e} = 0$, evaluated at $c^*(t)$ for the equilibrium level of t . If individuals negotiate the prices at which they trade, while the government controls the nominal money supply, then the equilibria are the subset of solutions to $\dot{e} = 0$ which also satisfy the condition for negotiated prices. In this section, we consider possible steady state equilibria when the government sets the nominal quantity of money and a particular pricing rule holds. The motivation for the pricing rule is discussed afterwards. In the next section, we consider equilibria with exogenous prices.

When a seller and buyer complete a deal, each of them has a surplus relative to his next best alternative, namely waiting to complete a deal with the next

trader encountered. Price negotiation determines the division of this surplus between buyer and seller. We assume that prices are chosen so that the utility gain to the buyer equals the utility gain to the seller.⁴ That is, the gain from the change of status from seller to buyer is equated with the gain from consumption less the loss from change of status from buyer to searcher for production opportunities

$$W_m - W_e = u + W_u - W_m \quad (15)$$

Combining (15) with the equation for the values of different positions, (8b), we see that the pricing rule implies (using (4))

$$r + b = s \quad \text{or} \quad r + \frac{f}{e} = \frac{f}{m} \quad (16)$$

That is, if we are in a steady state equilibrium satisfying the equal sharing of the trading surplus, then the rate of interest plus the arrival rate of buyers equals the arrival rate of sellers. Implicitly differentiating (16), we know that m and e are positively related by using (1) and (3):

$$\left. \frac{dm}{de} \right|_{r+b=s} = \frac{\frac{f_e}{m} + \frac{f - ef_e}{e^2}}{\frac{f_m}{e} + \frac{f - mf_m}{m^2}} > 0 \quad (17)$$

We assume that (16) appears as in Figure 1. That is, we assume that there exists a positive⁵ level of inventories, $\underline{e}$, which is the solution to

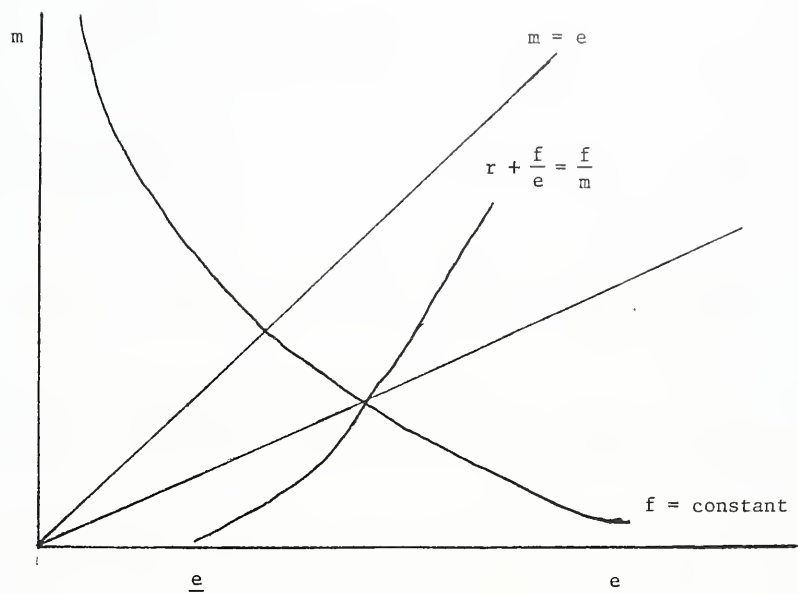
$$\lim_{m \rightarrow 0} f(\underline{e}, m)/m = r$$

We are assuming that the arrival rate of sellers becomes very small as inventories become small, even if the number of shoppers is much smaller. We also assume the existence of an m to satisfy (16) for every value of e greater than $\underline{e}$. If f is symmetric, the curve will lie below the 45° line.

⁴This is the Raiffa bargaining solution. See Luce and Raiffa Section 6.10.

⁵The analysis would be essentially unchanged with $\underline{e}$ equal to zero provided s went to zero in the neighborhood of the origin.

Figure 1



Using (16) to make m a function of e , we can examine optimal choice, c^* , and the steady state condition, $\dot{e}=0$, in a single diagram in (c^*, e) space. From Figure 1, we see that we will not have a solution for $e < \underline{e}$. At $e = \underline{e}$, $b = 0$. Thus t and c^* are both equal to zero. For $e > \underline{e}$,

$$t = \frac{1}{2} b = f/2e,$$

as can be seen from (10) and (16). Differentiating t with respect to e , using (17), we have

$$\left. \frac{dt}{de} \right|_{r+b=s} = \frac{2e(f_e + f_m \frac{dm}{de}) - 2f}{4e^2} \quad (18)$$

$$= \frac{1}{2} [ef_m + (e/m)^2(f - mf_m)]^{-1} [(ef_e - f)(\frac{f_m}{e} + \frac{f - mf_m}{m^2}) + ef_m(\frac{f_e}{m} + \frac{f - ef_e}{e^2})]$$

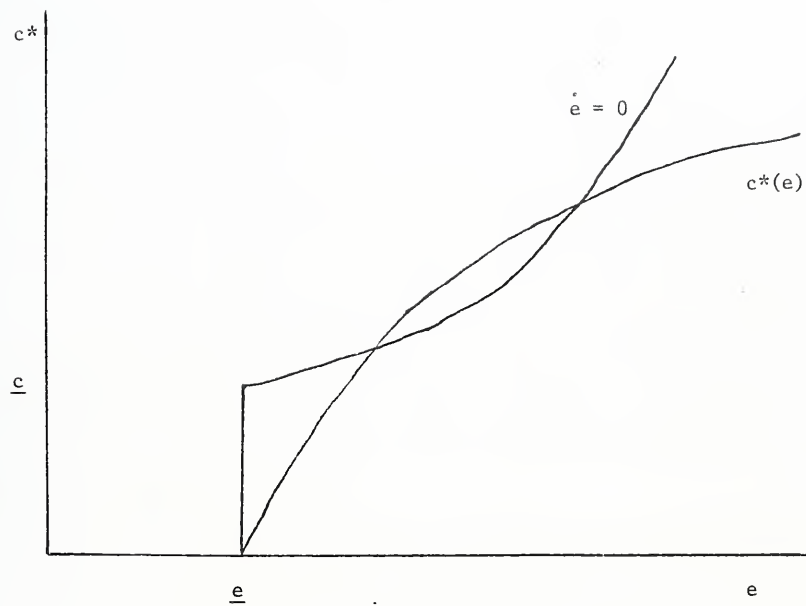
$$= \frac{1}{2} [ef_m + (e/m)^2(f - mf_m)]^{-1} (f/m^2)(ef_e + mf_m - f) > 0$$

Thus c^* increases with e . In addition, c^* has an upper bound, which is less than u .

Analyzing $\dot{e} = 0$, using (16) to relate m to e , we see that $\dot{e} = 0$ has the shape shown in Figure 2 - vertical above $\underline{e}$ to $\underline{c}$, c^* and e positively related above $\underline{c}$, and a vertical asymptote at a finite level of e , representing the maximal sustainable inventories when all projects are undertaken. Combining these curves in a single diagram, Figure 2, we see the possibility of multiple equilibria. When price setting satisfies the equal utility gain condition and the economy has the potential of operating at a positive level, there are multiple steady state equilibria. Those equilibria with higher willingness to produce have greater stocks of inventories and greater real money supplies (lower prices for a given nominal money supply.) There may be many more equilibria than the three shown in the diagram.

With constant returns rather than increasing returns in matching, (16) would imply a unique ratio of e to m , and so unique values of b and s , and therefore

Figure 2



c^* . Thus c^* would be a horizontal line in Figure 2, with a discontinuity at $c=\underline{c}$. There is then a unique equilibrium, ignoring the (now) uninteresting equilibrium at the origin.

Having described equilibrium with the equal utility gain assumption, it remains to justify its use. I shall not review the discussion of its appropriateness as an equilibrium concept in bargaining theory. Rather I will describe a peculiarity of the bargaining situation in this model and mention the difficulty with extensions of the model which would address them.

In an equilibrium where all goods sell at the same price and all money holdings are in single multiples of the price of a good, there must be sticky prices. The attempt to charge a higher price than the going price must fail (at least in the short run) since no buyer has more money. The attempt to bargain for a lower price is limited by the difficulty of estimating the ability to purchase of someone who has slightly less money than the going price; there are several consistent conjectures, giving different size ranges of equilibria. (Equilibria might not occur at a particular price since it remains necessary to have an adequate incentive for positive production.)

Thus, at first examination, bargaining theory appears capable of supporting many equilibria, not just those compatible with the equal utility gain assumption. To use the equal utility gain assumption without complication, equilibrium should have individuals with a continuum of money holdings so that there is a unique evaluation of the consequences of having slightly less or slightly more money. With a distribution of money holdings, however, bargaining theory implies a distribution of trading prices. Thus, the analysis required is much more complicated than before. In addition, in such a setting, some transactions will occur at a price equal to the entire money holdings of the buyer, since the utility possibility frontier may not cross the 45° line, further

complicating the analysis. With my inability, thus far, to satisfactorily solve a more complicated, more satisfactory equilibrium model, there are two alternatives with the simple model used here. One is to impose a pricing equation such as that used above. A second is to analyze equilibrium for arbitrary prices, to which we turn next. After this analysis we will consider the local efficiency properties of the fixed price equilibria and the subset of them that satisfy the equal utility gain condition.

7. Equilibrium With Price Controls

If the government controls prices, but not production decisions, the possible steady state equilibria are solutions to the equation $\dot{e}=0$, evaluated at $c^*(t)$. Substituting from (4), the equilibria are the simultaneous solutions to

$$aG(c^*)(n-e-m) = f(e,m) \quad (19)$$

$$c^* = c^*(t) = c^*\left(\frac{f^2}{rem+ef+mf}\right) \quad (20)$$

We will analyze the set of solutions to these equations, considering the case where f is homothetic, for concreteness.

We proceed by plotting (19) and (20) in a (c^*, e) diagram for given m ($0 < m < n$), and analyzing the shift in the curves when m shifts. Differentiating (19) we have

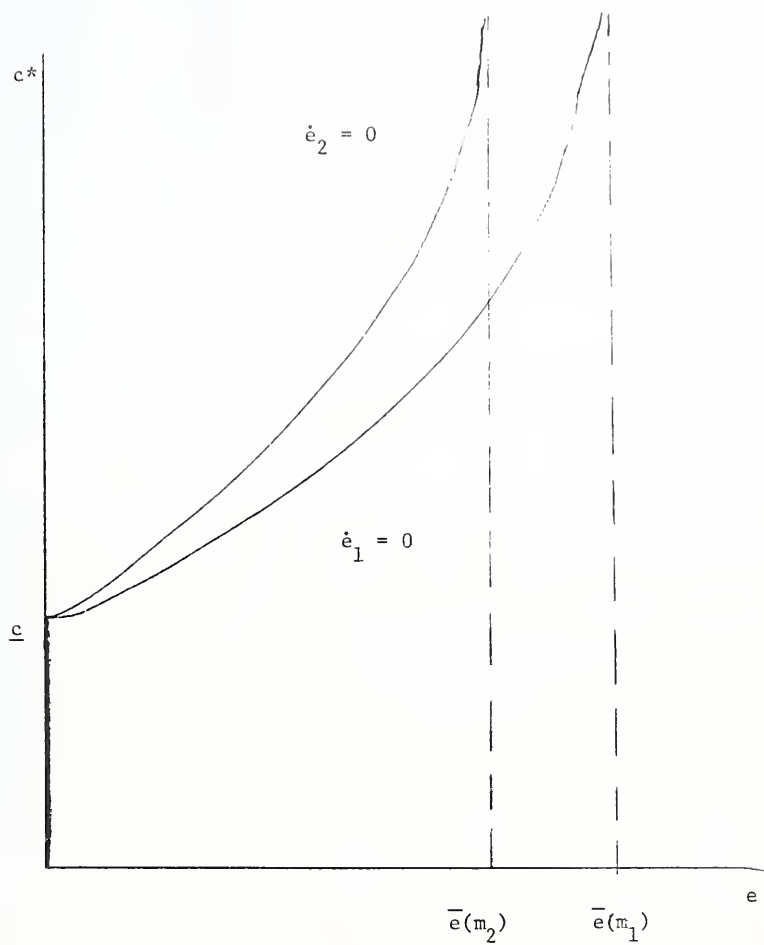
$$\left. \frac{\partial c^*}{\partial e} \right|_{\dot{e}=0} = \frac{f_e + aG}{aG'(n-e-m)} > 0 \quad (21)$$

$$\left. \frac{\partial c^*}{\partial m} \right|_{\dot{e}=0} = \frac{f_m + aG}{aG'(n-e-m)} > 0$$

Thus, along $\dot{e}=0$, c^* is positively related with e . For c^* below $\underline{c}$ (and so $G=0$), the only solution is $e=0$. As c^* rises without limit, G approaches 1 and e approaches its maximal sustainable level, $\bar{e}(m)$, given by

$$a(n-\bar{e}-m) = f(\bar{e}, m) \quad (22)$$

Figure 3



Loci where $\dot{e} = 0$

Differentiating (22) we have

$$\frac{\partial \bar{e}}{\partial m} = - \frac{a + \frac{f}{m}}{a + \frac{f}{e}} < 0 \quad (23)$$

Thus, for $m_2 > m_1 > 0$ we have the curves shown in Figure 3 and labeled $\dot{e}_2=0$, $\dot{e}_1=0$. Since G need not have nice properties, the curves have no necessary concavity.

To analyze (20), we can to consider the behavior of t , since c^* is increasing in t . First note that t and c^* are zero at e equal to 0 and plus infinity. Differentiating t , we have

$$\begin{aligned} \frac{\partial t}{\partial e} &= f[rem + ef + mf]^{-1} [(2rem + ef + mf)f_e - (rm + f)f] \\ &= mf[rem + ef + mf]^{-1} [(2r + b+s)ef_e - (r+s)f] \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{\partial t}{\partial m} &= f[rem + ef + mf]^{-1} [(2rem + ef + mf)f_m - (re + f)f] \\ &= ef[rem + ef + mf]^{-1} [(2r + b+s)mf_m - (r+b)f] \end{aligned} \quad (25)$$

We limit analysis to the case of a unique turning point of c^* as a function of e .

Combining (24) and (25), one has

$$e \frac{\partial t}{\partial e} + m \frac{\partial t}{\partial m} = em[rem + ef + mf]^{-1} (2r + b+s)(ef_e + mf_m - f) > 0 \quad (26)$$

Thus, if there were constant returns, one of the derivatives, (24) or (25), would be positive and the other negative. With the assumed increasing returns, both derivatives might be positive. Thus, assuming a unique turning point of c^* in e , we have the shapes shown in Figure 4 where $c_1^*(e)$ is drawn for value m_1 with $m_1 < m_2$. Thus, $c_2^*(e)$ crosses $c_1^*(e)$ to the left of the maximum of $c_1^*(e)$ and has a higher maximum.⁶

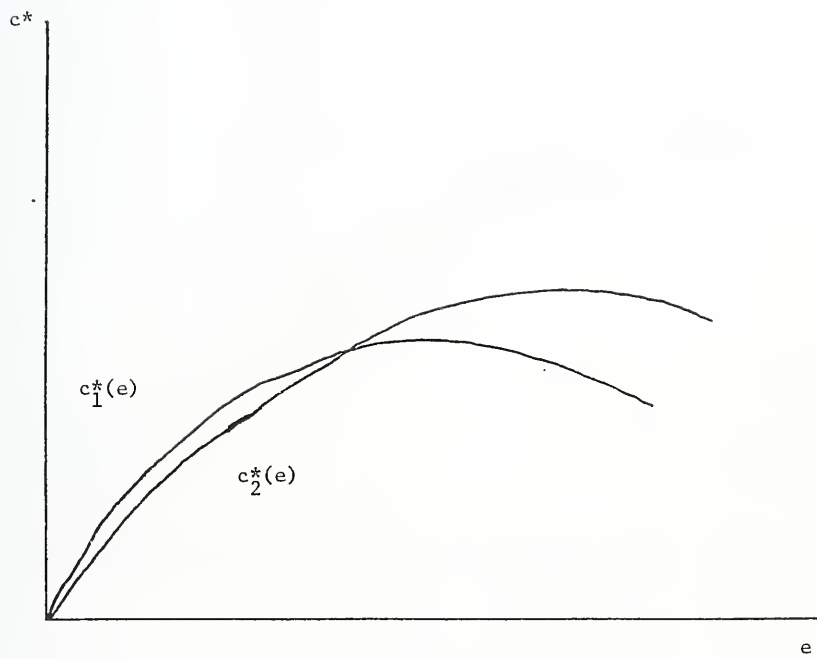
In Figure 5 we combine Figures 3 and 4 under the assumptions that the economy has equilibria other than the shut-down equilibrium and that these occur where c^* is rising in e and before the intersection of c_1^* and c_2^* . Thus there are at least three equilibria. There can be many more depending on the parameters. Equilibria at different m values are marked E_i , where $m_1 < m_2$. The picture can be quite different since the c^* curves can cross before some of the equilibria.

Assuming price controls and naive expectations it would be straightforward to analyze dynamics and so monetary policy for a policy of giving money to individuals without either money or inventories.⁷ More interesting dynamic assumptions would be much more difficult to analyze.

⁶ $c_2^*(\frac{em_2}{m_1}) > c_1^*(e)$ by homotheticity and increasing returns.

⁷ One would examine $\dot{e}$, (5), in (e, m) space, using $c^*(t)$.

Figure 4



8. Local Efficiency

An increase in the willingness to produce (real money held constant) makes it easier for others to buy but harder for them to sell. Thus the externalities associated with a change in production willingness can net out as positive or negative. A greater real money supply (willingness to produce held constant) implies a greater fraction of the population buying rather than selling or searching. This makes it easier to sell but harder to buy, again leaving no necessary sign on the net externality from increased real money. In this section we examine the determinants of the signs of these externalities. We do not analyze policies to bring about changes in c^* or m . With prices frozen, a change in c^* can be induced by subsidizing production, while a change in m is accomplished by giving money to some of the unemployed.

Denote by $W(e_0; m, c^*)$ the aggregate present discounted value of utility for an economy with an initial level of inventories e_0 and constant levels of real money, m , and willingness to produce, c^* . W equals the present discounted value of consumption, which has a flow utility level $uf(e, m)$ less the present discounted value of labor disutility, which has the flow value $a(n - e - m) \int_0^{c^*} cdG$.

Thus we have

$$W(e_0; m, c^*) = \int_0^{\infty} e^{-rt} [uf(e, m) - a(n - e - m) \int_0^{c^*} cdG] dt \quad (27)$$

$$\text{s.t. } \dot{e} = aG(c^*)(n - e - m) - f(e, m)$$

$$e(0) = e_0$$

It is straightforward to calculate the derivatives of W with respect to m and c^* , evaluated at a value of e_0 where $\dot{e}$ is zero (the calculations are in the Appendix). Differentiating, we have

$$\frac{\partial W}{\partial c^*} = \frac{aG'(c^*)(n - e - m)}{r} \left(\frac{uf_e + a \int_0^{c^*} cdG}{r + f_e + aG} - c^* \right) \quad (28)$$

$$\frac{\partial W}{\partial m} = \left(\frac{f_m + aG}{r} \right) \left(\frac{uf_m + a \int_0^{c^*} cdG}{f_m + aG} - \frac{uf_e + a \int_0^{c^*} cdG}{r + f_e + aG} \right) \quad (29)$$

We want to examine these derivatives at an equilibrium willingness to produce, c^* - where (11) holds. Rewriting (11) as

$$(r+t+aG)c^* = ut + a \int_0^{c^*} cdG \quad (30)$$

and substituting for $a \int_0^{c^*} cdG$ in (28) we have

$$\frac{\partial W}{\partial c^*} = \frac{aG'(n - e - m)}{r} \frac{(u - c^*)(f_e - t)}{(r + f_e + aG)} \quad (31)$$

We noted above, (13), that the private value of being in a position with inventories was $(u - c^*)t/r$. The social value of adding a permanent unit to inventories is the product of the increase in the flow of aggregate transactions, f_e , times the value of each transaction (net of replacement cost), $u - c^*$, divided by the interest rate. As in the discussion of W_e above, the actual search process for production opportunities is equivalent to instantaneous replacement at cost c^* . Thus the social value of changing the cutoff rule takes the familiar form of the difference between a social value and a private value. The remaining terms reflect the fact that we are evaluating a change in c^* , which affects the level of e differently over time.

Equation (31) holds at an equilibrium value of c^* and any value of m .

In an equilibrium satisfying the equal utility gain pricing condition, (16), we have

$$t = f/2e \quad (32)$$

Thus equation (31) implies

$$\text{sign } \frac{\partial W}{\partial c^*} = \text{sign } \left(f_e - \frac{f}{2e} \right) \quad (33)$$

That is, if it were possible to permanently alter willingness to produce without altering the real money supply, it would be socially worthwhile to raise the willingness to produce if the marginal product of inventories in the trade process exceeded half the average product.

We turn now to a change in the real money supply. The interpretation of (31) reflects the fact that, by their production decisions, individuals are choosing when to hold inventories rather than search (and search involves no externalities here). Since m is constant, this is the relevant margin for social evaluation of c^* as well. However, m is determined as a consequence of price setting, not by decisions to be money holders or not. The addition of money, and so a money holder, reduces both search and inventory holding, and the latter may also involve externalities. In addition, the stock of money holdings can be changed instantaneously, while, by changing c^* , the stock of inventories is altered according to the differential equation, (5). Thus, (29) takes the particular form which it does.

To analyze $\frac{\partial W}{\partial m}$ further, let us assume that $\frac{\partial W}{\partial c^*} < 0$. Then (substituting from (28) and (30) in (29)) we have

$$\begin{aligned} \frac{\partial W}{\partial m} &> \left(\frac{f_m + aG}{r} \right) \left(\frac{uf_m + a \int_0^{c^*} cdG}{f_m + aG} - c^* \right) \\ &= r^{-1} \left((u - c^*)(f_m - t) + rc^* \right) \end{aligned} \quad (34)$$

If the equilibrium also satisfies the equal utility gain pricing rule, we can conclude that

$$\frac{\partial W}{\partial c^*} < 0 \text{ implies } \frac{\partial W}{\partial m} > 0 \quad (35)$$

From nondecreasing returns in search, (2), and $f_e < f/2e$ (from (33)), we have

$$2f_m > f/m.$$

From the pricing rule, (16), the positivity of r , and the value of t , (32), we have

$$2f_m > f/m = r + f/e > f/e = 2t \quad (36)$$

Thus, in equilibrium, welfare is increasing in willingness to produce or real money (or possibly both) the other held constant.

With the government controlling m , but not c^* , analysis of an asymptotically optimal real money supply would have to evaluate the impact of changes of m on the time paths of b and s and so on c^* , the willingness to produce. If the government can control production as well as the real money supply, optimization will imply an asymptotic steady state satisfying $\partial W / \partial m = 0$ in (34). The asymptotically optimal money supply must reflect the fact that more money implies more time spent shopping. Supplying more money in this model is not a way of making nonliquid wealth liquid. Rather, more real money is more wealth and so more shopping and less production.

9. Velocity of Money

The instantaneous income velocity of money is $f(e,m)/m$, an endogenous variable.⁸ Across equilibria, those equilibria with higher e have higher m (see

⁸To explore the determination of velocity more thoroughly, one should add search intensity variables to the transactions technology. In addition, credit and the use of money for transactions in assets would affect the measured income velocity. Further complications could be introduced by disaggregating commodities and the transactions technology.

(17)). With increasing returns, at least one of f/m or f/e must then be larger. By the pricing equation, (16), both f/m and f/e move in the same direction. Thus velocity is higher at the equilibrium with the greater production level.

If the transactions technology is homothetic one can also conclude that the pricing equation, (16), appears as in Figure 1, crossing rays through the origin from below. To see this, let us first consider constant returns to scale. Then, the pricing condition, (16), is a ray through the origin. With homotheticity and increasing returns, $r + f/e - f/m$ has at most one zero on any ray, being first positive and then negative. Since (16) is positively sloped and comes out of the e axis, it can only cut rays from below.

10. An Alternative Formulation

In the model above there are three mutually exclusive activities, buying, selling, and searching for production activities. Additional real money in the economy increases the number of buyers and so decreases the numbers engaged in either search or selling. This result follows because someone with money prefers to buy and consume before searching again, rather than searching and selling followed by buying twice. The supply of real money has a second effect on the economy since a change in the number of shoppers changes the profitability of production and so the willingness to produce.

As an alternative formulation, assume that buying can take place simultaneously with selling or searching. (Think of couples.) Then individual workers divide their time between searching and selling. The division of time depends on the willingness to produce and the arrival rate of buyers. Individuals with more money have a lower willingness to produce. As a special case, if there is no willingness to produce when money holdings are

sufficient for a purchase ($c^* < \underline{c}$), then this alternative formulation is equivalent to the previous formulation. Thus it is appropriate to say that additional real money has a wealth effect, decreasing willingness to produce. In addition, by making sales easier and purchases harder, additional real money has a second effect on willingness to produce.

11. Varying Tastes

Thus far, it has been assumed that all goods are evaluated identically by consumers. We now assume that when a consumer observes a unit of output, he learns that the utility of consumption is u , an (independent) random draw from the distribution $H(u)$. We analyze this situation for a given exogenous price level.⁹ The focus of the analysis is to contrast the private cutoff rules, u^* and c^* , with their social counterparts. Thus, we repeat the analysis of Section 8 for this more general model. First, we quickly derive the equilibrium conditions.

The time path of inventories now reflects the fact that only the fraction $(1-H(u^*))$ of potential transactions are carried out. Thus, equation (5) becomes

$$\dot{e} = aG(c^*)(n-e-m) - f(e,m)(1-H(u^*)) \quad (37)$$

The three value equations, (7) become

$$\begin{aligned} rW_e &= b(1-H)(W_m - W_e) \\ rW_m &= s \int_{u^*}^{\infty} (u + W_u - W_m) dH(u) \\ rW_u &= a \int_0^{c^*} (W_e - W_u - c) dG(c) \end{aligned} \quad (38)$$

⁹Without this restriction, varying evaluations of goods would lead to varying negotiated trade prices and varying money holdings, greatly complicating the model. Prices set on a take-it-or-leave-it basis would be uniform if sellers did not vary in their money holdings. However, such an equilibrium suffers from the same stickiness as the negotiated equilibrium described above.

where $b(1-H)$ is the arrival rate of completed sales. The optimal cutoff rules, (6), are now

$$\begin{aligned} c^* &= W_e - W_u \\ u^* &= W_m - W_u \end{aligned} \quad (39)$$

Since W_m is larger than W_e , it follows immediately that u^* is larger than c^*

$$u^* - c^* = W_m - W_e = \frac{s \int_{u^*}^{\infty} (u - u^*) dH}{r + b(1-H)} > 0 \quad (40)$$

That is, in equilibrium, the (utility) shadow price determining flows from aggregate inventories exceeds the shadow price determining flows into inventories.¹⁰ For aggregate efficiency (given the transactions technology) these shadow prices should be equal, as we will see.

The aggregate present discounted value of utility, (27), now becomes

$$\begin{aligned} W(e_0; m, c^*, u^*) &= \int_0^{\infty} e^{-rt} [f(e, m) \int_{u^*}^{\infty} u dH - a(n - e - m) \int_0^{c^*} c dG] dt \\ \text{s.t. } \dot{e} &= aG(c^*)(n - e - m) - f(e, m)(1 - H(u^*)) \\ e(0) &= e_0 \end{aligned} \quad (41)$$

In dynamics programming terminology, we can rewrite (41) as

$$rW = f(e, m) \int_{u^*}^{\infty} u dH - a(n - e - m) \int_0^{c^*} c dG + \frac{\partial W}{\partial e} \dot{e} \quad (42)$$

¹⁰ This is not the case in the similar generalization of the barter model I have analyzed previously. In this way, the constant price money model is not equivalent to a barter model. For analysis of nonequivalence of money and barter models, see Kohn (1982).

Calculating the derivatives of W with respect to c^* and u^* at an equilibrium

where $\dot{e} = 0$ we have

$$\begin{aligned} r \frac{\partial W}{\partial c^*} &= a(n-e-m)G' \left(\frac{\partial W}{\partial e} - c^* \right) \\ r \frac{\partial W}{\partial u^*} &= fH' \left(\frac{\partial W}{\partial e} - u^* \right) \end{aligned} \quad (43)$$

where

$$\frac{\partial W}{\partial e} = \frac{f_e \int_{u^*}^{\infty} u dH + a \int_0^{c^*} c dG}{r + aG + (1-H)f_e} \quad (44)$$

Thus, optimization with respect to u^* and c^* would make them equal, giving the same shadow price for additions to and subtractions from inventory. In equilibrium, however, demanders act as if the price for purchases were higher than the price is for producers. Presumably this reflects the lack of interest on money.

12. Concluding Remarks

The model analyzed here has a cash-in-advance constraint or a finance constraint.¹¹ It seems likely that progress on understanding the roles of money and monetary policy will involve the use of some constraint along these lines. It would be possible to pursue the analysis of the optimal quantity of money in this model. Given the structure of the model, such an analysis would reflect the role of money in affecting the supply of labor for production (through either the time used for shopping or the wealth effect) as well as the profitability of production (through the ease of completing sales). The analysis would be very limited, however, by the primitive state of the model. Major extensions of the model would be required to accommodate nonsteady state equilibrium or equilibrium inflation (even at a constant rate) or credit. The absence of credit implicitly

¹¹ For equilibrium analysis with such a constraint see, e.g., Grandmont and Younes (1973).

assumes a sticky interest rate (of zero). It seems likely that credit with a sticky interest rate would not change the model significantly. Thus consideration of credit is likely to concentrate on the effects of a flexible interest rate. Without these elements (especially without credit) the analysis of monetary policy must be omitting some of the most important elements, and can not satisfactorily consider open market operations.

In a bargaining setting, it seems appropriate to describe prices as sticky if individuals who have met do not adjust prices to carry out mutually advantageous trades.¹² In the model as formulated in Section 6, there are no sticky prices in this sense nor any reason for sticky prices. One interesting project would be to reformulate the model so that there were reasons for sticky prices in this sense. These could come from the familiar sources of the implicit contract literature - insurance with limited observability, long term contracts, and fairness constraints. Sticky prices also come from prices that are offered on a take-it-or-leave-it basis to a heterogenous population (or more generally from bargaining failures, which naturally happen when bargaining games are specified). With such a reformulation one could examine how sticky prices change the properties of equilibrium. An easier project would be to impose sticky prices on this economy to examine the induced changes in comparative static properties.

The Arrow-Debreu model plays a central role in the analysis of micro-economists. Models of particular phenomena (externalities, noncompetitive behavior, missing markets) are generally constructed preserving all other features of the competitive general equilibrium model. This has been an extraordinarily fruitful way of organizing and conducting research into the range

¹²An alternative approach to sticky prices would be that with a change in the economy prices temporarily do not satisfy the equilibrium rule for price determination, even though no mutually advantageous deals are missed.

of microeconomic questions. Micro based models addressing macro questions have generally (implicitly or explicitly) followed the same approach. Examples are Lucas (1972) where separate markets, with a random division of suppliers, preserve their competitive properties, and Lucas and Prescott (1974) where search in the labor market is combined with a competitive output market. Yet, in the Arrow-Debreu model there is no explicit resource- or time-using trade coordination device. Thus, there is no natural transactions role for money. In addition, it is plausible that the difficulties of trade coordination are a major factor in the development of macro problems and in their response to government policies. If there is to be a satisfactory micro based theory of money and macro problems, it seems likely that it must dispense with the fictitious Walrasian auctioneer. This paper is a start on this task - the construction of micro based general equilibrium models with a natural transactions role for money and the possibility of macro unemployment difficulties. By building on this start, it may be possible to achieve a greater understanding of modern economies and to design more successful monetary and fiscal policies.

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Derivation of Welfare Derivatives

We derive equations (28) and (29), using the method spelled out more fully in Diamond (1980). From (27) we have

$$rW = uf(e,m) - a(n-e-m) \int_0^{c^*} cdG + \frac{\partial W}{\partial e} \dot{e} \quad (A1)$$

Differentiating and evaluating at $\dot{e} = 0$, we have

$$r \frac{\partial W}{\partial e} = u \frac{\partial f}{\partial e} + a \int_0^{c^*} cdG - \frac{\partial W}{\partial e} (aG(c^*) + \frac{\partial f}{\partial e}) \quad (A2)$$

$$r \frac{\partial W}{\partial m} = u \frac{\partial f}{\partial m} + a \int_0^{c^*} cdG - \frac{\partial W}{\partial e} (aG(c^*) + \frac{\partial f}{\partial m}) \quad (A3)$$

$$r \frac{\partial W}{\partial c^*} = -a(n-e-m)c^*G' + \frac{\partial W}{\partial e} (aG'(n-e-m)) \quad (A4)$$

Solving (A2) for $\frac{\partial W}{\partial e}$ and substituting in (A3) and (A4) we have (29) and (28).



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